Some Rules and some Examples about Sudoku

Stefano Sofia

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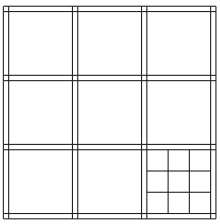
1 Definitions

Grid, Boxes and Cells: the grid is composed by 9 boxes, and each box is composed by 9 cells.



Unit: a group of 9 cells which must each contain a different digit in the solution. A unit can be a row, a column or a box

Digit: the numerical value which must be placed in the cells

Candidate: a possible solution for an unsolved cell. In a single cell each digit is a candidate (**example 1**), while in a unit for a given digit there can be many candidates (**example 2**) and (**example 3**)

Constraint: a group of candidates of which only one can be true

Unit Constraint: a constraint for a single digit within a unit

Link: a link is a connection between two cells in a unit that have a common digit

Strong Link: a link between two candidates when one of them must be true. If thefollowing logical deductions can be made for candidates A and B, they have a strong link:

IF A IS FALSE $\Rightarrow B$ MUST BE TRUE
IF B IS FALSE $\Rightarrow A$ MUST BE TRUE

Strong links can be present in

- bivalue cell, i.e. a single cell with only two candidates left (two candidates, two digits) (**example 1**)
- bilocation unit, i.e. unit constraint with only two candidates left for a single digit (two candidates, one single digit, see **Conjugate Pair**) (**example 3**)

For bivalue cells and bilocation units a strong link enforces a weak link

Weak Link: a weak link means that at most one of the two statements is true (**example 2**)

Conjugate Pair (CP): a pair of candidates for a single digit with a strong link, i.e. the last two candidates for a single digit in a unit (**example 3**)

Example 1: strong link in bivalue cells

9	**7**	3 5	2 4 8	3 8	3 4 5	2 8	**1**	**6**
4	**2**	1 5	6 8	1 6 8	5 9	8 9	**7**	**3**
6	**8**	1 3	2 7	1 3 7	3 9	2 9	**4**	**5**

In row 2 col 4 assume candidate **A** is digit 6 and candidate **B** is digit 8:

$$A \neq 6 \Rightarrow B = 8$$
$$B \neq 8 \Rightarrow A = 6$$

Same way for digits 2 and 9 in row 3 col 7.

Example 2: weak link

9	**7**	3 5	2 4 8	3 8	3 4 5	2 8	**1**	**6**
4	**2**	1 5	6 8	1 6 8	5 9	8 9	**7**	**3**
6	**8**	1 3	2 7	1 3 7	3 9	2 9	**4**	**5**

Assume candidate **A** is digit 8 in row 1 col 5 and candidate **B** is digit 8 in row 1 col 7:

$$A \neq 8 \nRightarrow B = 8$$
$$B \neq 8 \nRightarrow A = 8$$

since there are more than two cells that can be 8 in row 1 (row 1 col 4).

Example 3: strong link in bilocation units

9	**7**	3 5	2 4 8	3 8	3 4 5	2 8	**1**	**6**
4	**2**	1 5	 6 8	1 6 8	 5 9	 8 9	**7**	**3**
6	**8**	1 3	2 7	1 3 7	3 9	2 9	**4**	**5**

Assume candidate **A** is digit 9 in row 3 col 6 and candidate **B** is digit 9 in row 3 col 7:

$$A \neq 9 \Rightarrow B = 9$$
$$B \neq 9 \Rightarrow A = 9$$

Therefore 9 in row 3 col 6 and 9 in row 3 col 7 form a conjugate pair, because they are the last two candidates for digit 9 in row 3.

2 Basic Solving Techniques

2.1 Naked Singles

A single digit, and only this digit	is restricted to	a single cell

Example 4

		3	1	2 5	3	1	2 5	3
7	8	9			9	7	8	9
			1	2 5		1	2 5	
		9			9			9
	6			**4**		1 7	8	3 9

2.2 Hidden Singles

A single digit, which might be hidden among other digits	is restricted to	a single cell

Example 5

4	**7**	1 5 9
3	**8**	1 5 6 9
2	1 9	1 5 9

2.3 Locked Candidates

A single digit, which might be hidden among other digits	is restricted to	a single row of a box or a single column of a box

This type causes eliminations either

- in the row or column of the entire unit (**example 6**) or
- eliminations in the box (**example 7**)

Example 6

1 6	1 5 7 8	1 5 7	**9**	1 2 5 7 8	1 2 5 8	**4**	**3**	1 5 6 8
1 3 4	1 3 4 5 7 8	**2**	**6**	1 4 5 7 8	1 5 8	**9**	5 8	1 5 8
1 4 6	**9**	1 4 5	2 4 5	1 2 4 5 8	**3**	2 6 8	2 5 6 8	**7**

Example 7

4	1 5	**7**
2 3 5	**8**	1 3 5
6	1 2 5 9	1 5 9
5 7 9	1 4 5 7 9	1 4 5 6 9
2 7	**3**	1 8
2 5 9	2 5 9	5 6 8 9
1	4 5 7 9	3 4 5 9
3 5 7	**6**	3 4 5
8	4 5	**2**

2.4 Naked Pairs

Two digits, and only these two digits	are restricted to	two cells in a single row (**example 8**) or two cells in a single column or two cells in a single box

Example 8

7	1 2 4 5	1 4 5	2 4 5	**9**	6 8	6 8	1 6 8	**3**

2.5 Hidden Pairs

Two digits, which might be hidden among other digits	are restricted to	two cells in a single row (**example 9**) or two cells in a single column or two cells in a single box (**example 10**)

Example 9

8	2 5	**1**	2 7	3 5	**6**	3 7	**9**	**4**
3	2 5	4 6	2 4 7	1 5	**9**	1 6 7	**8**	1 2 7
9	**7**	4 6	2 4	**8**	1 3	**5**	2 6	1 2 3

Example 10

7	2 3	2 3 5
1 2 5 9	**8**	**4**
1 5 9	**6**	3 5

2.6 Naked Triples

Three digits, and only these three digits	are restricted to	three cells in a single row or three cells in a single column or three cells in a single box (**example 11**)

The cells which make up a Naked Triple don't have to contain every digit of the triple.

Example 11

1 6	1 5 7 8	1 5 7
1 3 4	1 3 4 5 7 8	**2**
1 4 6	**9**	1 4

2.7 Hidden Triples

Three digits, which might be hidden among other digits	are restricted to	three cells in a single row (**example 12**) or three cells in a single column or three cells in a single box (**example 13**)

The cells which make up a Hidden Triple don't have to contain every digit of the triple and these three digits.

Example 12

1 2	1 2 4 8	1 2 8 9	2 3 4 6 7	2 4 8	1 2 6 7 9	3 6 9	**5**	4 9

Example 13

9	4 6 8	3 6
3 5 6	5 6	**7**
2	1 4 6 8	1 3 4 6

2.8 Naked Quads

Four digits, and only these four digits	are restricted to	four cells in a single row or four cells in a single column or four cells in a single box (**example 14**)

The cells which make up a Naked Quad don't have to contain every digit of the quad.

Example 14

5 7 9	1 4 5 7	1 4 5 6
2 7	**3**	1 8
2 5 9	2 5	6 8

3 Medium Solving Techniques

3.1 X-Wing

Two CP related to a single digit, each of them located in a row (or a column)	are restricted to	exactly the same two columns (**example 15**) (or two rows)

Example 15

	4	3	9	8	6	2	5	6
6			4	2	5			
2				6	1	6	9	4
9	6	6			4	6	7	6
3			6		8			
4	1	6	2		9	6	6	3
8	2		5	6	6		6	6
	6	6		4				5
5	3	4	8	9	6	7	1	6

3.2 Swordfish

The Swordfish pattern is a variation on the "X-Wing" pattern above specified.

A single digit staying in three rows (or columns)	is restricted to	the same three columns (or rows)

The presence of the digit in each row (or in each column) can be restricted to a CP (respectively **example 16** and **example 17**), but this is not a necessary condition (**example 18**).

The X-Wing and Swordfish techniques can be further generalised to include rows containing digits restricted to four cells in the same four columns (called a Jellyfish, which is an advanced solving technique).

Example 16: two cells swordfish by row

	1		5	5		**8**	5	**3**
5				**9**	**6**	**1**		
		4	5	**8**	**1**	5	**6**	
9	5	5	**4**		**3**	5	5	**6**
	2	5		**6**		5	**1**	
			8		**5**			**7**
	6	5	5	**3**		**4**		**1**
			1	**7**		**6**	**3**	**5**
1	5	**3**	**6**	5			**2**	

Example 17: two cells swordfish by column

	1		5	5		**8**	5	**3**
5				**9**	**6**	**1**		
		4	5	**8**	**1**	5	**6**	
9	5	5	**4**		**3**	5	5	**6**
	2	5		**6**		5	**1**	
			8		**5**			**7**
	6	5	5	**3**		**4**		**1**
			1	**7**		**6**	**3**	**5**
1	5	**3**	**6**	5			**2**	

Example 18: three cells swordfish

		5	2	1	7		6	
7	1	9	4	6	3	2	7	5
2	6	7	9	5	8	7	1	7
4	7	6	1	8	2	7	5	7
			7	4	5	1		6
1	5	7	3	9	6	4	7	8
6	3	1	5	7				2
5	7	7	8	2	1	6	3	7
7	2	7	6	3		5	7	1

3.3 Two String Kite

Two CP related to a single digit, one located in a row and the other located in a column	are linked	in a single box

Note that

- the cells in both the CP must be in different boxes
- the cells in the common block must be distinct

Example 19: the first CP is in col 3 (9) and the second one is in row 9 (9).

			7	**8**	9	**4**		9
				9	9	9		9
9			**6**				**3**	
8	9	**5**	**4**	**6**	9	**7**	**1**	9
	9	9	9	**5**	9	9		9
	1	**4**	9	9	**8**	9		**6**
	6	9	9	9	9			**5**
					4	**6**	**9**	
	9	**7**	9	**2**	**6**			

Example 20: the first CP is in col 1 (4) and the second one is in row 2 (4).

2 3 4 5	2 4 5	**1**	**9**	2 3	**7**	5 6	4 5 6	**8**
6	2 4 9	2 9	**1**	**8**	**5**	**7**	**3**	2 4 9
2 3 5 8 9	2 5 8 9	**7**	**4**	**6**	2 3	**1**	5 9	2 5 9
1 2 5 8	**3**	**4**	2 6 7	**9**	2 6 8	5 6 8	1 5 6 7	1 5 6
1 2 8 9	2 6 7 8 9	2 6 8 9	**5**	2 3	**4**	3 6 8	1 6 7 9	1 3 6 9
5 8 9	5 6 7 8 9	6 8 9	3 6 7	**1**	3 6 8	**4**	**2**	3 5 6 9
2 4 8	2 4 6 8	**5**	2 3 6	**7**	**1**	**9**	4 6 8	3 4 6
2 9	**1**	2 3 6 9	**8**	**4**	2 3 6	3 5 6	5 6	**7**
7	4 6 8	3 6 8	3 6	**5**	**9**	**2**	1 4 6 8	1 3 4 6

Example 21: the first CP is in column 3 (9) and the second one is in row 4 (9).

4	**8**	**6**	**5**	**2**	**7**	**1**	**9**	**3**
		1			6 9	**5**	**8**	2 7
		7 9	3 8		1 8 9	**6**	2 7	**4**
6	1 5 9	**8**	4 7	4 7	5 9	**2**	**3**	1 5
		7 9	**1**	5 9	**3**	**4**	**6**	**8**
1 5	**4**	**3**	2 6	**8**	2 6	**7**	1 5	**9**
8	**6**	**5**	3 7	3 7	1 2	**9**	**4**	1 2
9	**7**	**4**	2 8	1 5		**3**	1 2	**6**
1 3	1 3	**2**	**9**	**6**	**4**	**8**	5 7	5 7

4 Advanced Solving Techniques

4.1 XY-Wing

Its name comes from the pattern formed by the numbers and it is not related to “X-Wing”.

Two couples of candidates (in bivalue cells), respectively $([XY], [XZ])$ and $([XY], [YZ])$	are linked	by being in the same unit

Then Z can be eliminated from the candidates of all cells that occupy the intersection of the units containing $[XZ]$ and $[YZ]$.

Examples of possible combinations:

	XY			XZ	

	YZ			*	

*	XY	*		XZ	
YZ			*	*	*

Example 22: X is 1, Y is 8 and Z is 2.

4	**8**	**6**	**5**	**2**	**7**	**1**	**9**	**3**
		1			6 9	**5**	**8**	2 7
		7 9	3 8		1 8	**6**	2 7	**4**
6	1 5 9	**8**	4 7	4 7	5 9	**2**	**3**	1 5
		7 9	**1**	5 9	**3**	**4**	**6**	**8**
1 5	**4**	**3**	2 6	**8**	2 6	**7**	1 5	**9**
8	**6**	**5**	3 7	3 7	1 2	**9**	**4**	1 2
9	**7**	**4**	2 8	1 5	1 2 5 8	**3**	1 2	**6**
1 3	1 3	**2**	**9**	**6**	**4**	**8**	5 7	5 7